Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for real numbers and simple algebraic expressions in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. knows, explains, and uses equivalent representations for rational numbers and simple algebraic expressions including integers, fractions, decimals, percents, and ratios; rational number bases with integer exponents; rational numbers written in scientific notation with integer exponents; time; and money (2.4.K1a) **($)**. 2. compares and orders rational numbers, the irrational number pi, and algebraic expressions (2.4.K1a) **($)**, e.g., which expression is greater $^{-}3n$ or 3n? It depends on the value of n. If n is positive, 3n is greater. If n is negative, $^{-}3n$ is greater. If n is zero, they are equal. 3. explains the relative magnitude between rational numbers, the irrational number pi, and algebraic expressions (2.4.K1a). 4. recognizes and describes irrational numbers (2.4.K1a), e.g., $\sqrt{2}$ is a non-repeating, non-terminating decimal; or π (pi) is a non-terminating decimal. 5. ▲ knows and explains what happens to the product or quotient when (2.4.K1a): a. a positive number is multiplied or divided by a rational number greater than zero and less than one, e.g., if 24 is divided by 1/3, will the answer be larger than 24 or smaller than 24? Explain. b. a positive number is multiplied or divided by a rational number greater than one, C c. a nonzero real number is multiplied or divided by zero, (For purposes of assessment, an explanation of division by zero will not be expected.) 6. explains and determines the absolute value of real numbers (2.4.K1a).	The student… 1. generates and/or solves real-world problems using equivalent representations of rational numbers and simple algebraic expressions (2.4.A1a) **($)**, e.g., a paper reports a company's gross income as $1.2 billion and their total expenses as $30,450,000. What is the company's net profit? 2. determines whether or not solutions to real-world problems using rational numbers, the irrational number pi, and simple algebraic expressions are reasonable (2.4.A1a) **($)**, e.g., the city park is putting a picket fence around their circular rose garden. The garden has a diameter of 7.5 meters. The planner wants to buy 20 meters of fencing. Is this reasonable?

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Teacher Notes: Number sense refers to one's ability to reason with numbers and to work with numbers in a flexible way. The ability to compute mentally, to estimate based on understanding of number relationships and magnitudes, and to judge reasonableness of answers are all involved in number sense.

The student with number sense will look at a problem holistically before confronting the details of the problem. The student will look for relationships among the numbers and operations and will consider the context in which the question was posed. Students with number sense will choose or even invent a method that takes advantage of their own understanding of the relationships between numbers and between numbers and operations, and they will seek the most efficient representation for the given task. Number sense can also be recognized in the students' use of benchmarks to judge number magnitude (e.g., 2/5 of 49 is less that half of 49), to recognize unreasonable results for calculations, and to employ non-standard algorithms for mental computation and estimation. (Developing Number Sense: Addenda Series, Grades 5-8, NCTM, 1991)

If you think about division as undoing multiplication, you'll see why we don't divide by zero. EXAMPLE: Try to divide 245 ÷ 0. To divide, think of the related multiplication equation. 245 ÷ 0 = ? asks the same question as ? x 0 = 245. When you think about it this way, you can see that you're really stuck because any number times zero is zero! So, mathematicians say that division by zero is undefined. Math On Call *226*. *Don't try to divide by Zero.*

At this grade level, real numbers include positive and negative numbers and very large numbers (one billion) and very small numbers (one-millionth). **Relative magnitude** refers to the size relationship one number has with another – is it much larger, much smaller, close, or about the same? For example, using the numbers 219, 264, and 457, answer questions such as:

- Which two are closest? Why?
- Which is closest to 300? To 250?
- About how far apart are 219 and 500? 5,000?
- If these are 'big numbers,' what are small numbers? Numbers about the same? Numbers that make these seem small?

(Elementary and Middle School Mathematics, John A. Van de Walle, Addison Wesley Longman, Inc., 1998)

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **($)**. The National Standards in Personal Finance are included in the Appendix.

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Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of the real number system; recognizes, applies, and explains their properties; and extends these properties to algebraic expressions.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. explains and illustrates the relationship between the subsets of the real number system [natural (counting) numbers, whole numbers, integers, rational numbers, irrational numbers] using mathematical models (2.4.K1a), e.g., number lines or Venn diagrams. 2. ▲ identifies all the subsets of the real number system [natural (counting) numbers, whole numbers, integers, rational numbers, irrational numbers] to which a given number belongs (2.4.K1l). (For the purpose of assessment, irrational numbers will not be included.) 3. names, uses, and describes these properties with the rational number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) **($)**: a. commutative, associative, distributive, and substitution properties [commutative: a + b = b + a and ab = ba; associative: a + (b + c) = (a + b) + c and a(bc) = (ab)c; distributive: a(b + c) = ab + ac; substitution: if a = 2, then 3a = 3 x 2 = 6]; b. identity properties for addition and multiplication and inverse properties of addition and multiplication (additive identity: a + 0 = a, multiplicative identity: a • 1 = a, additive inverse: $^{+}5 + {}^{-}5 = 0$, multiplicative inverse: 8 x 1/8 = 1); c. symmetric property of equality, e.g., 7 + 2 = 9 has the same meaning as 9 = 7 + 2; d. addition and multiplication properties of equalities, e.g., if a = b, then a + c = b + c; e. addition property of inequalities, e.g., if a > b, then a + c > b + c; f. zero product property, e.g., if ab = 0, then a = 0 and/or b = 0.	The student… 1. generates and/or solves real-world problems with rational numbers using the concepts of these properties to explain reasoning (2.4.A1a) **($)**: a. ▲ commutative, associative, distributive, and substitution properties; e.g., we need to place trim around the outside edges of a bulletin board with dimensions of 3 ft by 5 ft. Explain two different methods of solving this problem and why the answers are equivalent. b. ▲ identity and inverse properties of addition and multiplication; e.g., I had $50. I went to the mall and spent $20 in one store, $25 at a second store and then $5 at the food court. To solve: [$50 – ($20 + $25 + $5) = $50 - $50 = 0]. Explain your reasoning. c. symmetric property of equality; e.g., Sam took a $15 check to the bank and received a $10 bill and a $5 bill. Later Sam took a $10 bill and a $5 bill to the bank and received a check for $15. $15 = $10 + $5 is the same as $10 + $5 = $15 d. addition and multiplication properties of equality; e.g., the total price (P) of a car, including tax (T), is $14, 685. 33. If the tax is $785.42, what is the sale price of the car (S)? e. zero product property, e.g., Jenny was thinking of two numbers. Jenny said that the product of the two numbers was 0. What could you deduct from this statement? Explain your reasoning

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	2. analyzes and evaluates the advantages and disadvantages of using integers, whole numbers, fractions (including mixed numbers), or decimals in solving a given real-world problem (2.4.A1a) **($)**, e.g., in the store everything is 33 1/3% off. When calculating the discount, which representation of 33 1/3% would you use and why?

Teacher Notes: From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y, x + y = y + x.
- Property of an equation: For all numbers a, b, and c, if a = b, then a + c = b + c.
- Property of an inequality: For all numbers a, b, and c, if a > b, then a – c > b – c.

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 3: Estimation – The student uses computational estimation with real numbers in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. estimates real number quantities using various computational methods including mental math, paper and pencil, concrete objects, and/or appropriate technology (2.4.K1a) **($)**. *2.* uses various estimation strategies and explains how they were used to estimate real number quantities and simple algebraic expressions (2.4.K1a) **($)**. 3. knows and explains why a decimal representation of the irrational number pi is an approximate value (2.4.K1c). 4. knows and explains between which two consecutive integers an irrational number lies (2.4.K1a).	The student… 1. adjusts original rational number estimate of a real-world problem based on additional information (a frame of reference) (2.4.A1a) **($)**, e.g., estimate the height of a building from a picture. In another picture, a person is standing next to the building. By using the person as a frame of reference adjust your original estimate. 2. estimates to check whether or not the result of a real-world problem using rational numbers and/or simple algebraic expressions is reasonable and makes predictions based on the information (2.4.A1a) **($)**, e.g., you have a $4,000 debt on a credit card. You pay the minimum of $30 per month. Is it reasonable to pay off the debt in 10 years? 3. determines a reasonable range for the estimation of a quantity given a real-world problem and explains the reasonableness of the range (2.4.A1c) **($)**, e.g., determine the reasonable range for the weight of a book and explain why this range is reasonable. 4. determines if a real-world problem calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental mathematics, paper and pencil, concrete objects, and/or appropriate technology (2.4.A1a) **($)**, e.g., do you need an exact or an approximate answer when calculating the area of the walls in a room to determine the number of rolls of wallpaper needed to paper the room?. An approximation is appropriate for the area but an exact answer is needed for the number of roles. What would you do if you were wallpapering 2 rooms?

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	5. explains the impact of estimation on the result of a real-world problem (underestimate, overestimate, range of estimates) (2.4.A1a) **($)**, e.g., you are estimating the total of three large purchases ($489, $553, and $92). If you rounded each to the nearest $10, would your estimate be slightly lower or higher than the actual amount spent? If you rounded each to the nearest $100, would your estimate be slightly lower or higher than the actual amount spent?

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Teacher Notes: Estimate, as a verb, means to make an educated guess based on information in a problem or to give an answer close to the exact number. Estimation is used when an exact answer is not needed, as in many real-life situations for which "ballpark" computations are acceptable. Good number sense enables one to estimate a quantity, estimate a measure, or estimate an answer.

Estimation serves as an important companion to computation. It provides a tool for judging the reasonableness of computational methods including mental math, paper and pencil, concrete objects, and appropriate technology. However, being able to compute does not automatically lead to an ability to estimate or judge reasonableness of answers. Frequent modeling by the teacher helps students develop a range of estimation strategies. Students should be encouraged to frequently explain their thinking as they estimate. As with exact computation, sharing estimation strategies allows students access to others' thinking and provides opportunities for class discussion. Identifying the estimation strategy by name is not critical; however, explaining the thinking behind the strategy to make a valid estimation is important. (Principles and Standards for School Mathematics, NCTM, 2000)

Mental math and **estimation** are distinct but related mathematical skills. Proficiency in mental math contributes to increased skill in estimation. In order for students to become more familiar with estimation, teachers should introduce estimation with examples where rounded or estimated numbers are used. Emphasis should be placed on real-world examples where only estimation is required, e.g., About how many hours do you sleep a night? Using the language of estimation is important, so students begin to realize that a variety of estimates (answers) are possible.
In addition, when students are taught specific estimation strategies, they develop mental math and estimation skills easier. Estimation strategies include front-end with adjustment, compatible "nice" numbers, clustering, special numbers, or truncation.

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator.

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Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 4: Computation – The student models, performs, and explains computation with rational numbers, the irrational number pi, and algebraic expressions in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) **($)**. 2. performs and explains these computational procedures with rational numbers (2.4.K1a): a. **▲N** addition, subtraction, multiplication, and division of integers b. **▲N** order of operations (evaluates within grouping symbols, evaluates powers to the second or third power, multiplies or divides in order from left to right, then adds or subtracts in order from left to right); c. approximation of roots of numbers using calculators; d. multiplication or division to find: i. a percent of a number, e.g., what is 0.5% of 10? ii. percent of increase and decrease, e.g., if two coins are removed from ten coins, what is the percent of decrease? iii. percent one number is of another number, e.g., what percent of 80 is 120? iv. a number when a percent of the number is given, e.g., 15% of what number is 30? e. addition of polynomials, e.g., (3x – 5) + (2x + 8). f. simplifies algebraic expressions in one variable by combining like terms or using the distributive property (2.4.K1a), e.g., $^{-}3(x - 4)$ is the same as $^{-}3x + 12$. 3. finds factors and common factors of simple monomial expressions (2.4.K1d), e.g., given the monomials $10m^2n^3$ and $15a^2mn^2$ some common factors would be 5m, $5mn^2$, and n^2.	The student… 1. ▲ generates and/or solves one- and two-step real-world problems using computational procedures and mathematical concepts (2.4.A1a) with **($)**: a. ■ rational numbers, e.g., find the height of a triangular garden given that the area to be covered is 400 square feet with a base of 12½ feet; b. the irrational number pi as an approximation, e.g., before planting, a farmer plows a circular region that has an approximate area of 7,300 square feet. What is the radius of the circular region to the nearest tenth of a foot? c. applications of percents, e.g., sales tax or discounts. (For the purpose of assessment, percents greater than or equal to 100% will NOT be used).

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Teacher Notes: Efficiency and accuracy means that students are able to compute single-digit numbers with fluency. Students increase their understanding and skill in addition, subtraction, multiplication, and division by understanding the relationships between addition and subtraction, addition and multiplication, multiplication and division, and subtraction and division. Students learn basic number combinations and develop strategies for computing that makes sense to them. Through class discussions, students can compare the ease of use and ease of explanation of various strategies. In some cases, their strategies for computing will be close to conventional algorithms; in other cases, they will be quite different. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and these invented approaches often can be used with efficiency and accuracy. (Principles and Standards for School Mathematics, NCTM, 2000)

The definition of computation is finding the standard representation for a number. For example, 6 + 6, 4 x 3, 17 – 5, and 24 ÷ 2 are all representations for the standard representation of 12. **Mental math** is mentally finding the standard representation for a number – calculating in your head instead of calculating using paper and pencil or technology. One of the main reasons for teaching mental math is to help students determine if a computed/calculated answer is reasonable; in other words, using mental math to estimate to see if the answer makes sense. Students develop mental math skills easier when they are taught specific strategies. Mental math strategies include counting on, doubling, repeated doubling, halving, making tens, multiplying by powers of ten, dividing with tens, finding fractional parts, thinking money, and using compatible "nice" numbers.

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

Technology is changing mathematics and its uses. The use of technology including calculators and computers is an important part of growing up in a complex society. It is not only necessary to estimate appropriate answers accurately when required, but also it is also important to have a good understanding of the underlying concepts in order to know when to apply the appropriate procedure. Technology does not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation. However, dividing a 5-digit number by a 2-digit number is appropriate with the exception of dividing by 10, 100, or 1,000 and simple multiples of each.

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Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains the general rule of a pattern from a variety of situations.

Eighth Grade Knowledge Base Indicators

The student…

1. identifies, states, and continues a pattern presented in various formats including numeric (list or table), algebraic (symbolic notation), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written using these **attributes**:
   a. counting numbers including perfect squares, cubes, and factors and multiples with positive rational numbers (number theory) (2.4.K1a).
   b. rational numbers including arithmetic and geometric sequences (arithmetic: sequence of numbers in which the difference of two consecutive numbers is the same, geometric: a sequence of numbers in which each succeeding term is obtained by multiplying the preceding term by the same number) (2.4.K1a), e.g., $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$;
   c. geometric figures (2.4.K1h);
   d. measurements (2.4.K1a);
   e. things related to daily life **($)**;
   f. variables and simple expressions, e.g., 1 – x, 2 – x, 3 – x, 4 – x, ...; or x, x^2, x^3, ...
2. generates and explains a pattern (2.4.K1a).
3. generates a pattern limited to two operations (addition, subtraction, multiplication, division, exponents) when given the rule for the nth term (2.4.K1a), e.g., the n^{th} term is n^2+1, find the first 4 terms beginning with n = 1; the terms are 2, 5, 10, and 17.
4. states the rule to find the nth term of a pattern using explicit symbolic notation (2.4.K1a), e.g., given 2, 5, 8, 11, …; find the rule for the nth term, the rule is 3n –1.

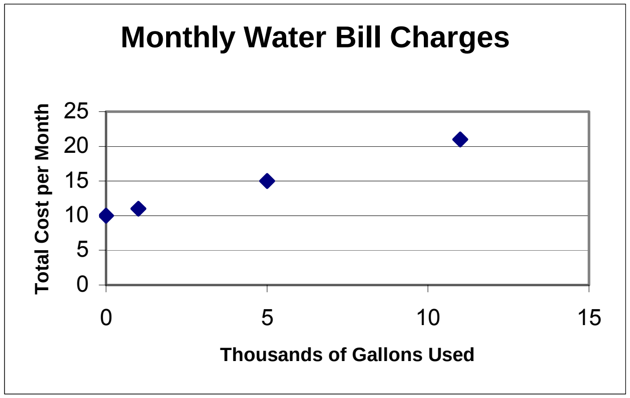
Eighth Grade Application Indicators

The student…

1. generalizes numerical patterns using algebra and then translates between the equation, graph, and table of values resulting from the generalization (2.4.A1d-e,j) **($)**, e.g., water is billed at $1.00 per 1,000 gallons, plus a basic fee of $10 per month for being connected to the water district.

1,000 Gallons	Cost in a given month		Graph these:
1	$10 + 1*1.00	→	(1, 11)
2	$10 + 2*.1.00	→	(2, 12)
.	.		
.	.		
n	10 + n*1.00	→	(n, 1.00n + 10)

where C = total cost and G = gallons used,
C = 1.00 G + 10]



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5. describes the pattern when given a table of linear values and plots the ordered pairs on a coordinate plane (2.4.K1f-g), e.g., in the table below, the pattern could be described as the x-coordinates are increasing by three, while the y-coordinates are increasing by 6, or the x is doubled and one is added to find the y.	2. recognizes the same general pattern presented in different representations [numeric (list or table), visual (picture, table, or graph), and written] (2.4.A1a,j) **($)**.

X	2	5	8	11
Y	5	11	17	23

Teacher Notes: A fundamental component in the development of classification, number, and problem solving skills is inventing, discovering, and describing patterns. Patterns pervade all of mathematics and much of nature. All **patterns** are either repeating or growing or a variation of either or both. Translating a pattern from one medium to another to find two patterns that are alike, even though they are made with different materials, is important so students can see the relationships that are critical to repeating patterns. With growing patterns, students not only extend patterns, but also look for a generalization or an algebraic relationship that will tell them what the pattern will be at any point along the way.

Working with **patterns** is an important process in the development of mathematical thinking. Patterns can be based on geometric attributes (shapes, regions, angles); measurement attributes (color, texture, length, weight, volume, number); relational attributes (proportion, sequence, functions); and affective attributes (values, likes, dislikes, familiarity, heritage, culture). (Learning to Teach Mathematics, Randall J. Souviney, Macmillan Publishing Company, 1994)

In the pattern that begins with 3, 5, 7, and 9; the explicit rule is 2n +1 and the recursive rule is add 2 to the previous term. Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

This process (working with patterns) can be used to develop or deepen understandings of important concepts in number theory, rational numbers, measurement, geometry, probability, and functions. Working with patterns provides opportunities for students to recognize, describe, extend, develop, and explain.

Number theory is the study of the properties of the counting numbers (positive integers), their relationships, ways to represent them, and patterns among them. Number theory includes the concepts of odd and even numbers, factors and multiples, primes and composites, greatest common factor and least common multiple, and sequences.

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THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 2: Variable, Equations, and Inequalities – The student uses variables, symbols, real numbers, and algebraic expressions to solve equations and inequalities in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. identifies independent and dependent variables within a given situation. 2. simplifies algebraic expressions in one variable by combining like terms or using the distributive property (2.4.K1a), e.g., $^{-}3(x - 4)$ is the same as $^{-}3x + 12$. 3. solves (2.4.K1a,e) **($)**: a. ▲ one- and two-step linear equations in one variable with rational number coefficients and constants intuitively and/or analytically; b. one-step linear inequalities in one variable with rational number coefficients and constants intuitively, analytically, and graphically; c. systems of given linear equations with whole number coefficients and constants graphically. 4. knows and describes the mathematical relationship between ratios, proportions, and percents and how to solve for a missing monomial or binomial term in a proportion (2.4.K1c), e.g., $\frac{2}{5} = \frac{1}{x + 2}$. 5. represents and solves algebraically **($)**: a. the number when a percent and a number are given, b. what percent one number is of another number, c. percent of increase or decrease, e.g., the price of a loaf of bread is $2.00. With a coupon, the cost is $1.00. What is the percent of decrease? 6. evaluates formulas using substitution **($)**.	The student… 1. represents real-world problems using (2.4.A1d) **($)**: a. ▲ ■ variables, symbols, expressions, one- or two-step equations with rational number coefficients and constants, e.g., today John is 3.25 inches more than half his sister's height. If J = John's height, and S = his sister's height, then J = 0.5S + 3.25. b. one-step inequalities with rational number coefficients and constants, e.g., after Randy paid $38.50 for a watch, he did not have enough money to by a calculator for $5.50. Represent this situation with an inequality. c. systems of linear equations with whole number coefficients and constants, e.g., two students collected the same amount of money for a walk-a-thon. One student received $5 per mile and a donation of $10, while the other student received $2 per mile and a donation of $20. How many miles did they walk? 2. solves real-world problems with two-step linear equations in one variable with rational number coefficients and constants and rational solutions intuitively, analytically, and graphically (2.4.A1e) e.g., Mike and Albert are friends, but Joe and Albert are not friends. Which of the following diagrams can be used to describe this situation? (Three dots labeled J, M, A: there is a line between J and M and line between M and A, but no line between J and A.) 3. generates real-world problems that represent (2.4.A1d) **($)**: a. one- or two-step linear equations, **($)**, e.g., given the equation 2x + 10 = 30, the problem could be I bought two shirts and a pair of pants for $10. How much was a shirt, if the total bill was $30? b. one-step linear inequalities, e.g., write a real-world situation that represents the inequality x + 10 > 30. The problem could be: If you give me $10, I will have more than $30.

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	4. explains the mathematical reasoning that was used to solve a real-world problem using one- or two-step linear equations and inequalities and discusses the advantages and disadvantages to various strategies that may have been used to solve the problem, (2.4.A1d) **($)**, e.g., given the inequality x + 10 > 30, subtract the same number from both sides of the inequality or graph as y1 = x + 10 and y = 30 and find on the graph where y1 is less than y2. The first method gives an exact answer; the second method is a visual representation and can be used to solve more difficult inequalities.

Teacher Notes: Understanding the **concept of variable** is fundamental to algebra. Students use various symbols, including letters and geometric shapes to represent unknown quantities that both do and do not vary. Quantities that are not given and do not vary are often referred to as unknowns or missing elements when they appear in equations, e.g., 2 + Δ = 4 or 3 • s = 15 where a triangle and s are used as variables. Various symbols or letters should be used interchangeably in equations.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **($)**. The National Standards in Personal Finance are included in the Appendix.

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Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 3: Functions – The student recognizes, describes, and analyzes constant, linear, and nonlinear relationships in a variety of situations.

Eighth Grade Knowledge Base Indicators

The student…

1. recognizes and examines constant, linear, and nonlinear relationships using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or appropriate technology (2.4.K1a,e-g) **($).**
2. knows and describes the difference between constant, linear, and nonlinear relationships (2.4.K1g).
3. explains the concepts of slope and x- and y-intercepts of a line (2.4.K1g).
4. recognizes and identifies the graphs of constant and linear functions (2.4.K1g) **($).**
5. identifies ordered pairs from a graph, and/or plots ordered pairs using a variety of scales for the x- and y-axis (2.4.K1g).

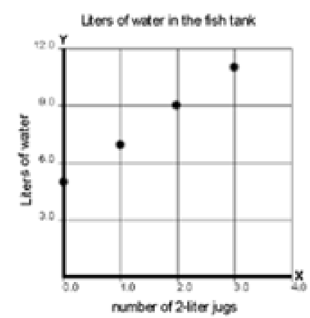
Eighth Grade Application Indicators

The student…

1. represents a variety of constant and linear relationships using written or oral descriptions of the rule, tables, graphs, and symbolic notation (2.4.A1d-f) **($).**
2. interprets, describes, and analyzes the mathematical relationships of numerical, tabular, and graphical representations (2.4.A1j) **($).**
3. ▲ translates between the numerical, tabular, graphical, and symbolic representations of linear relationships with integer coefficients and constants (2.4.A1a), e.g., a fish tank is being filled with water with a 2-liter jug. There are already 5 liters of water in the fish tank. Therefore, you are showing how full the tank is as you empty 2-liter jugs of water into it. Y = 2x + 5 (symbolic) can be represented in a table (tabular) –

X	0	1	2	3
Y	5	7	9	11

and as a graph (graphical) –



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Teacher Notes: Functions are relationships or rules in which each member of one set is paired with one, and only one, member of another set (an ordered pair). The concept of function can be introduced using function machines. Any number put in the machine will be changed according to some rule. A record of inputs and corresponding outputs can be maintained in a two-column format. Function tables, input/output machines, and T-tables may be used interchangeably and serve the same purpose.

Function concepts should be developed from **growing patterns**. Each term in a number sequence is related to its position in the sequence – the functional relationship. The pattern – 4, 7, 10, 13, 16, 19, and so on – is an arithmetic sequence *with a difference of 3*. The pattern could be described as *add 3* meaning that 3 must be added to the previous term to find the next. This pattern is explained by using the recursive definition for a sequence. The recursive definition for a sequence is a statement or a set of statements that explains how each successive term in the sequence is obtained from the previous term(s).

In the pattern 1, 4, 9, 16, 25, …, 225; there is *no common difference.* This sequence is not arithmetic or geometric (no common ratio between geometric terms). Neither is it a combination of the two; however, there is a pattern and the missing terms between 25 and 225 can be found. To find the term value, square the number of the term. The next missing terms would be 36, 49, 64, 81, 100, 121, and 144. This pattern is explained by using the explicit formula for a sequence. The explicit formula for a sequence defines a rule for finding each term in the number sequence related to its position in the sequence. In other words, to find the term value, square the number of the term – the 5^{th} term is 5^2, the 8^{th} term is 8^2, …

Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

In the Cartesian Coordinate System, the expression $x = 3$ cannot represent a function. When you graph all points where x = 3 you get a vertical line, so more than one y-value exists for the x-value x = 3. By definition, a function must have only one output value for any given input. For more information, consult a reference book under the topic, vertical line test.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 4: Models – The student generates and uses mathematical models to represent and justify mathematical relationships found in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. knows, explains, and uses mathematical models to represent and explain mathematical concepts, procedures, and relationships. Mathematical models include: a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, and mathematical relationships and to solve equations (1.1.K1-6, 1.2.K1, 1.2.K3, 1.3.K1-2, 1.3.K4, 1.4.K1-2, 2.1.K1a-b, 2.1.K1d-e, 2.1.K2-4, 2.2.K2-3, 3.1.K9, 3.2.K1-4, 3.3.K1-4, 3.4.K4, 4.2.K4-5) **($)**; b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures **($)**; c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.3.K3, 2.3.K6) **($)**: d. factor trees to model least common multiple, greatest common factor, and prime factorization (1.4.K3); e. equations and inequalities to model numerical relationships (2.2.K3, (3.4.K2) **($)**; f. function tables to model numerical and algebraic relationships (2.1.K5, 3.4.K2) **($)** ; g. coordinate planes to model relationships between ordered pairs and linear equations and inequalities (2.1.K5, 2.3.K1-5, 3.4.K2-3) **($)**;	The student… 1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include: a. process models (concrete objects, pictures, diagrams, flowcharts, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, mathematical relationships, and problem situations and to solve equations (1.1.A1-2, 1.2.A1-2, 1.3.A1-5, 1.4.A1, 2.1.A1, 3.1.A1, 3.2.A1-2, 3.3.A1, 3.4.A1-2) **($)**; b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to model problem situations **($)**; c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (3.2.A3) **($)**; d. equations and inequalities to model numerical relationships (2.1.A2, 2.2.A1-2, 2.3.A1, 3.4.A2) **($)**; e. function tables to model numerical and algebraic relationships (2.1.A2, 2.3.A2, 3.4.A2) **($)**; f. coordinate planes to model relationships between ordered pairs and linear equations and inequalities (2.3.A1 3.4.A2) **($)**; g. two- and three-dimensional geometric models (geoboards, dot paper, nets, or solids) and real-world objects to model perimeter, area, volume, surface area and properties of two- and three-dimensional figures (3.3.A3, 3.4.A2); h. scale drawings to model large and small real-world objects (3.1.A1-2, 3.3.A4);

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h. two- and three-dimensional geometric models (geoboards, dot paper, nets, or solids) and real-world objects to model perimeter, area, volume, surface area, and properties of two-and three-dimensional figures (2.1.K1c, 3.1.K1-6, 3.1.K8, 3.1.K10, 3.2.K5, 3.3.K4-5); i. scale drawings to model large and small real-world objects (3.3.K3-4); j. geometric models (spinners, targets, or number cubes), process models (coins, pictures, or diagrams), and tree diagrams to model probability (4.1.K1-5) **($)**; k. frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single and double stem-and-leaf plots, scatter plots, box-and-whisker plots, and histograms to organize and display data (4.2.K1, 4.2.K6) **($)**; l. Venn diagrams to sort data and to show relationships (1.2.K2).	i. geometric models (spinners, targets, or number cubes), process models (coins, pictures, or diagrams), and tree diagrams to model probability (4.1.A1-4); j. frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single and double stem-and-leaf plots, scatter plots, box-and-whisker plots, and histograms to describe, interpret, and analyze data (2.1.A1-2, 2.3.A2-3, 4.2.A1, 4.2.A3, 4.2.A1-3) **($)**; k. Venn diagrams to sort data and to show relationships. 2. ▲ determines if a given graphical, algebraic, or geometric model is an accurate representation of a given real-world situation **($).** 3. uses the mathematical modeling process to analyze and make inferences about real-world situations **($).**

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Teacher Notes: For assessment purposes, the mathematical modeling process appropriate to the indicator may be included as part of the item being assessed.

The **mathematical modeling** process involves:

a. selecting key features and relationships within the real-world situation and representing these concepts in mathematical terms through some sort of mathematical model,
b. performing manipulations and mathematical procedures within the mathematical model,
c. interpreting the results of the manipulations within the mathematical model,
d. using these results to make inferences about the original real-world situation.

The use of mathematical models is necessary for conceptual understanding. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. As students begin to use multiple representations of the same situation, they begin to develop an understanding of the advantages and disadvantages of various representations/models.

Many **mathematical models** are listed in this benchmark. The indicator lists some of the mathematical models that could be used to teach a concept. Each indicator in this benchmark is linked to other indicators in other benchmarks; those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3. In addition, the indicator in the other benchmarks identifies, in parentheses, the Models' indicator. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models).

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Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric figures and compares their properties in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. recognizes and compares properties of two- and three-dimensional figures using concrete objects, constructions, drawings, appropriate terminology, and appropriate technology (2.4.K1h). 2. discusses properties of triangles and quadrilaterals related to (2.4.K1h): a. sum of the interior angles of any triangle is 180°; b. sum of the interior angles of any quadrilateral is 360°; c. parallelograms have opposite sides that are parallel and congruent, opposite angles are congruent; d. rectangles have angles of 90°, sides may or may not be equal; e. rhombi have all sides equal in length, angles may or may not be equal; f. squares have angles of 90°, all sides congruent; g. trapezoids have one pair of opposite sides parallel and the other pair of opposite sides are not parallel; h. kites have two distinct pairs of adjacent congruent sides. 3. recognizes and describes the rotational symmetries and line symmetries that exist in two-dimensional figures (2.4.K1h), e.g., draw a picture with a line of symmetry in it. Explain why it is a line of symmetry. 4. recognizes and uses properties of corresponding parts of similar and congruent triangles and quadrilaterals to find side or angle measures using standard notation for similarity (~) and congruence (≅) (2.4.K1h). 5. knows and describes Triangle Inequality Theorem to determine if a triangle exists (2.4.K1h).	The student… 1. solves real-world problems by (2.4.A1a): a. ▲ ■ using the properties of corresponding parts of similar and congruent figures, e.g., scale drawings, map reading, proportions, or indirect measurements. b. applying the Pythagorean Theorem, e.g., indirect measurements, map reading/distance, or diagonals.

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6. ▲ uses the Pythagorean theorem to (2.4.K1h): a. determine if a triangle is a right triangle, b. find a missing side of a right triangle where the lengths of all three sides are whole numbers. 7. recognizes and compares the concepts of a point, line, and plane. 8. describes the intersection of plane figures, e.g., two circles could intersect at no point, one point, two points, or all points. 9. describes and explains angle relationships: a. when two lines intersect including vertical and supplementary angles; b. when formed by parallel lines cut by a transversal including corresponding, alternate interior, and alternate exterior angles. 10. recognizes and describes arcs and semicircles as parts of a circle and uses the standard notation for arc (⌒) and circle (⊙) (2.4.K1h).	

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Teacher Notes: Geometry is the study of shapes, their properties, and their relationships to other shapes. Symbols and numbers are used to describe their properties and their relationships to other shapes. The fundamental concepts in geometry are point (no dimension), line (one-dimensional), plane (two-dimensional), and space (three-dimensional). Plane figures are referred to as two-dimensional. Solids are referred to as three-dimensional. The base, in terms of geometry, generally refers to the side on which a figure rests. Therefore, depending on the orientation of the solid, the base changes.

From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y, x + y = y + x.
- Property of an equation: For all numbers a, b, and c, if a = b, then a + c = b + c.
- Property of an inequality: For all numbers a, b, and c, if a > b, then a – c > b – c.

The application of the Knowledge Indicators from the Geometry Benchmark, Geometric Figures and Their Properties are most often applied within the context of the other Geometry Benchmarks - Measurement and Estimation, Transformational Geometry, and Geometry From an Algebraic Perspective - rather than in isolation.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 2: Measurement and Estimation – The student estimates, measures, and uses geometric formulas in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. determines and uses rational number approximations (estimations) for length, width, weight, volume, temperature, time, perimeter, area, and surface area using standard and nonstandard units of measure (2.4.K1a) **($).** 2. selects and uses measurement tools, units of measure, and level of precision appropriate for a given situation to find accurate real number representations for length, weight, volume, temperature, time, perimeter, area, surface area, and angle measurements (2.4.K1a) **($).** 3. converts within the customary system and within the metric system. 4. estimates the measure of a concrete object in one system given the measure of that object in another system and the approximate conversion factor (2.4.K1a), e.g., a mile is about 2.2 kilometers; how far is 2 miles? 5. uses given measurement formulas to find (2.4.K1h): a. area of parallelograms and trapezoids; b. surface area of rectangular prisms, triangular prisms, and cylinders; c. volume of rectangular prisms, triangular prisms, and cylinders. 6. recognizes how ratios and proportions can be used to measure inaccessible objects (2.4.K1c), e.g., using shadows to measure the height of a flagpole. 7. calculates rates of change, e.g., speed or population growth.	The student… 1. solves real-world problems (2.4.A1a) by **($)**: a. converting within the customary and the metric systems, e.g., James added 30 grams of sand to his model boat that weighed 2 kg before it sank. With the sand included, what is the total weight of his boat? b. finding perimeter and area of circles, squares, rectangles, triangles, parallelograms, and trapezoids; e.g., Jane jogs on a circular track with a radius of 100 feet. How far would she jog in one lap? c. finding the volume and surface area of rectangular prisms, e.g., how much paint would be needed to cover a box with dimensions of 3 feet by 4 feet by 5 feet? 2. estimates to check whether or not measurements or calculations for length, weight, volume, temperature, time, perimeter, area, and surface area in real world problems are reasonable and adjusts original measurement or estimation based on additional information (a frame of reference) (2.4.A1a) **($)**, e.g., to check your calculation in finding the area of the floor in the kitchen; you count how many foot-square tiles there are on the floor. 3. uses ratio and proportion to measure inaccessible objects (2.4.A1c), e.g., using the length of a shadow to measure the height of a flagpole.

Teacher Notes: The term *geometry* comes from two Greek words meaning "earth measure." **Measurement** provides the tools required to apply geometric concepts in the real-world. **Estimation in measurement** is defined as making guesses as to the exact measurement of an object without using any type of measurement tool. Estimation helps students develop a relationship between the different sizes of units of measure. It helps students develop basic properties of measurement and it gives students a tool to determine whether a given measurement is reasonable.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 3: Transformational Geometry – The student recognizes and applies transformations on geometric figures in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. identifies, describes, and performs single and multiple transformations [reflection, rotation, translation, reduction (contraction/shrinking), enlargement (magnification/growing)] on a two-dimensional figure (2.4.K1a). 2. describes a reflection of a given two-dimensional figure that moves it from its initial placement (preimage) to its final placement (image) in the coordinate plane over the x- and y-axis (2.4.K1a,i). 3. draws (2.4.K1a): a. three-dimensional figures from a variety of perspectives (top, bottom, sides, corners); b. a scale drawing of a two-dimensional figure; c. a two-dimensional drawing of a three-dimensional figure. 4. determines where and how an object or a shape can be tessellated using single or multiple transformations (2.4.K1a).	The student… 1. generalizes the impact of transformations on the area and perimeter of any two-dimensional geometric figure (2.4.A1a), e.g., enlarging by a factor of three triples the perimeter (circumference) and multiplies the area by a factor of nine. 2. describes and draws a two-dimensional figure after undergoing two specified transformations without using a concrete object. 3. investigates congruency, similarity, and symmetry of geometric figures using transformations (2.4.A1g). 4. uses a scale drawing to determine the actual dimensions and/or measurements of a two-dimensional figure represented in a scale drawing (2.4.A1h).

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Teacher Notes: Transformational geometry is another way to investigate and interpret geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology. Some **transformations**, like translations, reflections, and rotations, do not change the figure itself. Other transformations, like reduction (contraction/shrinking) or enlargement (magnification/growing), change the size of a figure, but not the shape (congruence vs. similarity).

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 4: Geometry from an Algebraic Perspective – The student uses an algebraic perspective to examine the geometry of two-dimensional figures in a variety of situations.

Eighth Grade Knowledge Base Indicators

The student…

1. uses the coordinate plane to (2.4.K1a):
   a. ▲ list several ordered pairs on the graph of a line and find the slope of the line;
   b. ▲ recognize that ordered pairs that lie on the graph of an equation are solutions to that equation;
   c. ▲ recognize that points that do not lie on the graph of an equation are not solutions to that equation;
   d. ▲ determine the length of a side of a figure drawn on a coordinate plane with vertices having the same x- or y-coordinates;
   e. solve simple systems of linear equations.
2. uses a given linear equation with integer coefficients and constants and an integer solution to find the ordered pairs, organizes the ordered pairs using a T-table, and plots the ordered pairs on a coordinate plane (2.4.K1e-g).
3. examines characteristics of two-dimensional figures on a coordinate plane using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or other appropriate technology (2.4.A1g).

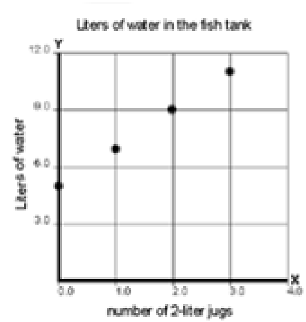
Eighth Grade Application Indicators

The student…

1. represents, generates, and/or solves distance problems (including the use of the Pythagorean theorem, but not necessarily the distance formula) (2.4.A1a), e.g., a student lives five miles west and three miles north of school and another student lives 4 miles south and 7 miles east of school. What is the shortest distance between the students' homes (as the crow flies)?
2. translates between the written, numeric, algebraic, and geometric representations of a real-world problem (2.4.A1a,d-g), e.g., given a situation: make a T-table, define the algebraic relationship, and graph the ordered pairs. The T-table can be represented as –

X	0	1	2	3
Y	5	7	9	11

as an algebraic relationship – 2 χ = 5,



and as a graph (graphical)

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Teacher Notes: A **number line** (a mathematical model) is a diagram that represents numbers with equal distances marked off as points on a line, and is an example of one-to-one correspondence (a relation). A number line can be used as a visual representation of numbers and operations. In addition, a number line used horizontally and vertically is a precursor to the coordinate plane; and the distance between two numbers on a number line is a precursor to absolute value.

A **coordinate plane** (coordinate grid) consists of a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis. These two lines intersect at a point called the origin. The x-axis and the y-axis divide the plane into four sections called quadrants. Any point on the coordinate plane can be named with two numbers called coordinates. The first number is the x-coordinate. The second number is the y-coordinate. Since the pair is always named in order (first x, then y), it is called an ordered pair.

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Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions, generate convincing arguments, and make predictions and decisions including the use of concrete objects in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. knows and explains the difference between independent and dependent events in an experiment, simulation, or situation (2.4.K1j) **($)**. 2. identifies situations with independent or dependent events in an experiment, simulation, or situation (2.4.K1j), e.g., there are three marbles in a bag. If you draw one marble and give it to your brother, and another marble and give it to your sister, are these independent events or dependent events? 3. ▲ finds the probability of a compound event composed of two independent events in an experiment, simulation, or situation (2.4.K1j), e.g., what is the probability of getting two heads, if you toss a dime and a quarter? 4. finds the probability of simple and/or compound events using geometric models (spinners or dartboards) (2.4.K1j). 5. finds the odds of a desired outcome in an experiment or simulation and expresses the answer as a ratio (2/3 or 2:3 or 2 to 3) (2.4.K1j). 6. describes the difference between probability and odds.	The student… 1. conducts an experiment or simulation with independent or dependent events including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions and make predictions about future events (2.4.A1i-j). 2. analyzes the results of an experiment or simulation of two independent events to generate convincing arguments, draw conclusions, and make predictions and decisions in a variety of real-world situations (2.4.A1i-j). 3. compares theoretical probability (expected results) with empirical probability (experimental results) in an experiment or simulation with a compound event composed of two independent events and understands that the larger the sample size, the greater the likelihood that the experimental results will equal the theoretical probability (2.4.A1i). 4. makes predictions based on the theoretical probability of (2.4.A1a,i): a. ▲ ■ a simple event in an experiment or simulation, b. compound events composed of two independent events in an experiment or simulation.

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Teacher Notes: Ideas from **probability** reinforce concepts in the other Standards, especially Number and Computation and Geometry. Students need to develop an intuitive concept of chance – whether or not something is unlikely or likely to happen. Probability experiences should be addressed through the use of concrete objects (process models); spinners, number cubes, or dartboards (geometric models); and coins (money models). Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results or outcomes of experiments. Some examples of uses of probability in every day life include: There is a 50% chance of rain today. What is the probability that the team will win every game?

Odds is a ratio of favorable to unfavorable outcomes whereas probability is a ratio of favorable outcomes to all possible outcomes. Probability can be written as a fraction, a decimal, and a percent, whereas odds can only be written as a ratio, e.g., 2/3, 2:3, or 2 to 3.

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Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (rational) and non-numerical data sets in a variety of situations.

Eighth Grade Knowledge Base Indicators	Eighth Grade Application Indicators
The student… 1. organizes, displays and reads quantitative (numerical) and qualitative (non-numerical) data in a clear, organized, and accurate manner including a title, labels, categories, and rational number intervals using these **data displays** (2.4.K1k) **($)**: a. frequency tables; b. bar, line, and circle graphs; c. Venn diagrams or other pictorial displays; d. charts and tables; e. stem-and-leaf plots (single and double); f. scatter plots; g. box-and-whiskers plots; h. histograms. 2. recognizes valid and invalid data collection and sampling techniques. 3. ▲ determines and explains the measures of central tendency (mode, median, mean) for a rational number data set (2.4.K1a). 4. determines and explains the range, quartiles, and interquartile range for a rational number data set (2.4.K1a). 5. explains the effects of outliers on the median, mean, and range of a rational number data set (2.4.K1a). 6. makes a scatter plot and draws a line that approximately represents the data, determines whether a correlation exists, and if that correlation is positive, negative, or that no correlation exists (2.4.K1k).	The student… 1. uses data analysis (mean, median, mode, range) in real-world problems with rational number data sets to compare and contrast two sets of data, to make accurate inferences and predictions, to analyze decisions, and to develop convincing arguments from these **data displays** (2.4.A1j) **($)**: a. frequency tables; b. bar, line, and circle graphs; c. Venn diagrams or other pictorial displays; d. charts and tables; e. stem-and-leaf plots (single and double); f. scatter plots; g. box-and-whiskers plots; h. histograms. 2. explains advantages and disadvantages of various data collection techniques (observations, surveys, or interviews), and sampling techniques (random sampling, samples of convenience, biased sampling, or purposeful sampling) in a given situation (2.4.A1j) **($)**. 3. recognizes and explains (2.4.A1j): a. misleading representations of data; b. the effects of scale or interval changes on graphs of data sets. 4. recognizes faulty arguments and common errors in data analysis.

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Teacher Notes: Graphs (data displays) are pictorial representations of mathematical relationships, are used to tell a story, and are an important part of statistics. When a graph is made, the axes and the scale (numbers running along a side of the graph) are chosen for a reason. The difference between numbers from one grid line to another is the interval. The interval will depend on the lowest and highest values in the data set. Emphasizing the importance of using equal-sized pictures or intervals is critical to ensuring that the data display is accurate.

Graphs take many forms:

- bar graphs and pictographs compare discrete data,
- frequency tables show how many times a certain piece of data occurs,
- circle graphs (pie charts) model parts of a whole,
- line graphs show change over time,
- Venn diagrams show relationships among sets of objects,
- line plots show frequency of data on a number line,
- single stem-and-leaf plots (closely related to line plots except that the number line is usually vertical and digits are used rather than x's) show frequency distribution by arranging numbers (stems) on the left side of a vertical line with numbers (leaves) on the right side,
- scatter plots show the relationship between two quantities,
- box-and-whisker plots are visual representations of the five-number summary – the median, the upper and lower quartiles, and the least and greatest values in the distribution – therefore, the center, the spread, and the overall range are immediately evident by looking at the plot, and
- histograms (closely related to stem-and-leaf plots) describe how data falls into different ranges.

Two important aspects of data are its *center* and its *spread.* The mean, median, and mode are **measures of central tendency** (averages) that describe where data are centered. Each of these measures is a single number that describes the data. However, each does it slightly differently. The **range** describes the spread (dispersion) of data. The easiest way to measure spread is the range, the difference between the greatest and the least values in a data set. Quartiles are boundaries that break the data into fourths.

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